CityWindmills
SMALL WIND TURBINES FOR URBAN USE

City Windmills' CEO explains wind power mathematics

In an earlier paper we discussed how energy derives from wind speed, and how compression due to the roof-top effect significantly enhances wind speed and therefore energy derived from a wind turbine.

This paper goes further and explains the relationship between average wind speeds at a location, and what power is actually generated, based on wind variability (i.e. calm to gusts..). **The mathematical reality shows that actually twice as much power is derived from a wind turbine than an average wind speed would suggest…**

We start with the basic equation :

Power generation from wind speed V is governed by a simple equation :-

Power = ½ x Area x $Velocity^3$ x Rho where Rho is Air density (≈1.25)

Or P(V) = $f(V^3)$ (Power generated is a function of wind-speed cubed)

Therefore power generated by a wind turbine is a function of area of the turbine struck by the wind, but more importantly by the cubed value of the velocity. Hence doubling wind speed will multiply the power generated by a turbine by a factor of 8!!

Simplistically :- let's assume an average 5m/s – simply, half the time we have a wind speed of 0 m/s, half the time at 10 m/s, (defined for now as the maximum wind speed)

at 5m/s average, the Power generated is P(5)
at 0m/s average, the Power generated is P(0) = 0xP(5) = 0
at 10m/s average, the Power generated is P(10) = P(2 x 5) = 2^3xP(5)

therefore 5m/s generates P(5),
but an average 5m/s wind generates = (P(0)) + P(10))/2
= (0xP(5) + 8xP(5))/2
= 4xP(5)

So actually, an average wind speed of 5 m/s simplistically generates 4x the power of a constant 5 m/s wind.
(N.B. if we assume average based on 1/3 x each of 0m/s, 5m/s, 10m/s the result yields 3x the power of a constant 5m/s wind.)

Expanding the above to smooth out a range of all wind speeds from 0-10 m/s, we see a convergence of 2x the power generated (See Table A)

Ok simplistic; let's go mathematical…

Theoretically :- so let's assume an even distribution of wind above 0 m/s, with power generated as a function of V^3. (V = wind speed). We integrate the cubic curve to derive the area under this curve, (i.e. total power generated) which yields an area $V^4/4$, defined over the distribution of wind speeds. As a formula :-

$$
\begin{aligned}
P(Vmean) &= P((\int_0^V (Vmax^3))/ Vmax)) \\
&= P(Vmax^4/4 / Vmax) = P(Vmax^3)/4 \\
\text{But } Vmax &= 2x\ Vmean \\
\text{Therefore } Pmean &= 8 \times P(Vmean^3)/4 \\
\text{or } Pmean &= 2 \times P(Vmean^3)
\end{aligned}
$$

ie 2x energy is derived from wind for a given average wind, based on our initial formula.

<u>In summary, although a given wind scenario tells us that a constant average wind in a location should generate a certain amount of power from a wind turbine, the mathematical reality is that twice the amount of power will actually be generated, based on more power derived from gusts at higher speeds.</u>

David Mapley, B.Sc.(Econ.), M.Sc., M.B.A.
CEO, City Windmills Holdings PLC
www.city-windmills.com
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Table A

Wind Speed (m/s)	Power (kwh)	2	3	5	10	20
0	0	0	0	0	0	0
1	1					1
2	8				8	8
3	27					27
4	64				64	64
5	125			125		125
6	216				216	216
7	343					343
8	512				512	512
9	729					729
Mean Wind 10	1000		1000	1000	1000	1000
11	1331					1331
12	1728				1728	1728
13	2197					2197
14	2744				2744	2744
15	3375			3375		3375
16	4096				4096	4096
17	4913					4913
18	5832				5832	5832
19	6859					6859
20	8000	8000	8000	8000	8000	8000
Sum		8000	9000	12500	24200	44100
Average		4000	3000	2500	2420	2205
vs Mean		4.0	3.0	2.5	2.4	2.2

Vs The Mean Wind Speed, as a factor, converges to 2…

We are actually summing a serious of cubes, then averaging.

The sum of a series of n numbers, cubed $= n^2 \times (n+1)^2 / 4$

We divide by n data points for the average $= n(n+1)^2/4$

The average speed is (n/2), therefore power $= n^3/8$

As n → ∞, this step-up factor $= (n(n+1)^2)/4 \div n^3/8$

$= 2((n+1)/n)^3$

Which converges to 2 as n → ∞